

## Module overview

This module explores the pedagogical and practical challenges that confront teachers as they help pupils to use ICT effectively in solving problems and learning mathematics. In this module we look at contrasting uses of the computer: a 'microworld' to explore, a generic 'thinking tool' and a didactical tool. Through planning, implementing and reflecting on lessons using the computer, the role and significance of ICT in mathematics and its relationship to the traditional curriculum are explored.

This guide is intended for use alongside the *Bowland Maths DVD* or website, which include a short introductory video for each of the activities; longer videos of lessons and teacher discussions and links to all the handouts and ICT-based problems. **Please ensure that the two software 'microworlds' are installed and tested on participants' computers before the session** – see the *Resources index* for this module on the DVD or website.

### Introductory session

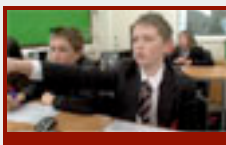
1 hour



- Explore using ICT as a "microworld"
- Reflect on the uses of ICT in your school
- Observe another teacher using ICT
- Plan a lesson using ICT

### Into the classroom

1 hour



- Introduce the microworld to the class
- Pupils explore the microworld and generate problems
- Pupils describe what is happening and share problems to explore
- Pupils work on the problems
- Pupils report back and share findings











### Follow-up session

1 hour




- Reflect on the *Spirolaterals* or *Dance Moves* lesson
- Explore the use of ICT as a thinking tool: *Magazine Sales*
- Observe a teacher using *Magazine Sales*
- Relate ICT to the new Programme of Study for Mathematics
- Look at the use of ICT in the Case Studies
- Consider using other resources to provoke thinking (optional)

## Resources Needed

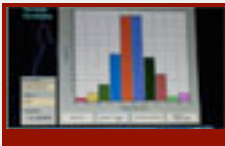
- |   |           |   |
|---|-----------|---|
|  | Handout 1 | Two microworlds to explore                  |
|  | Software  | Spirolaterals 'microworld'                  |
|  | Software  | Dance moves 'microworld'                    |
|  | Handout 2 | Roles of ICT in the maths classroom         |
|  | Handout 3 | A checklist for using ICT                   |
|  | Handout 4 | Making and selling a magazine               |
|  | Handout 5 | Spreadsheet hints                           |
|  | Handout 6 | Mathematics Programme of Study: Key stage 3 |
|  | Handout 7 | Notes on the problems                       |
|  | Handout 8 | Suggested further reading                   |




**Activity 2****Reflect on the uses of ICT in your school****10 minutes**

 [Handout 2](#) proposes three categories for the roles ICT can play in school mathematics:

- A **microworld** providing a rich, motivating domain to explore (e.g. *Spirolaterals*, *Dance Moves* and other specific 'applets')
- A **thinking tool** for working on Mathematics (e.g. graphics calculator, spreadsheet, database, graph plotter, dynamic geometry)
- A **didactic tool** that attempts to explain mathematics and develop fluency in routine skills in more motivating ways. (E.g. electronic textbook, *PowerPoint* presentation, computer games to develop fluency with multiplication tables)



Refer to  [Handout 2](#) and discuss the following questions:

- Which microworlds, thinking tools, and didactic tools do *you* use in mathematics?
- What relative importance do you give to each of the 3 roles in your lessons?
- How far do your pupils regard ICT as a *natural medium for doing mathematics*?  
(Or is the most powerful mathematical tool still the back of an envelope?)

You may like to watch Rob, Peter and Christine as they discuss the ways they use ICT in their school.

**Activity 3****Observe another teacher using ICT****20 minutes**

Now watch Rob using *Spirolaterals* with his class.



- How does the teacher help pupils engage with the situation?
- What resources other than computers are made available to pupils?  
How are these used?
- What problems do the pupils create?
- How does the teacher encourage pupils to stop 'playing' and start thinking?
- What potential can you see for developing mathematics in this lesson?

Later, you may also like to watch Christine as she uses the software *Dance moves*.

**Activity 4****Plan a lesson using ICT****15 minutes**

Now it's your turn to plan a lesson using *Spirolaterals* or *Dance Moves*.

Discuss your answers to the following questions

- What is the best way to introduce the activity to pupils?
- How will you ensure that pupils progress from "playing around" to working systematically and recording their thinking?
- When and how will you enable pupils to share their ideas and strategies?

On [Handout 3](#), you will find some more general suggestions and advice on planning lessons which use ICT.

This is the end of the *Introductory session*. After you have tried out your lesson with your own pupils, return for the *Follow-up session*.

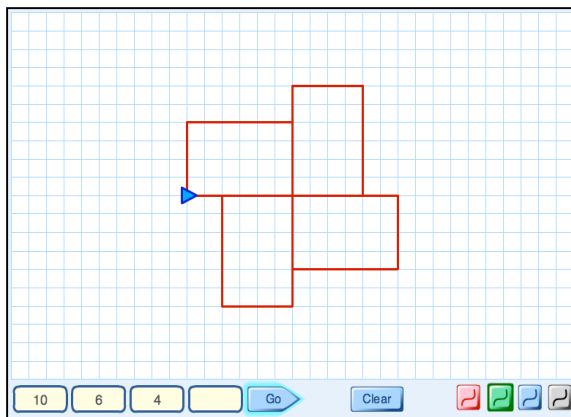
Resources to support the lessons, and suggested lesson plans, can be found in the *Into the classroom* session.

The following suggestions describe one possible approach to using the microworlds *Spirolaterals* or *Dance Moves* with your pupils. Each microworld will need between one and two lessons. Although the plans are presented together, we do advise, that you only use one program at a time!

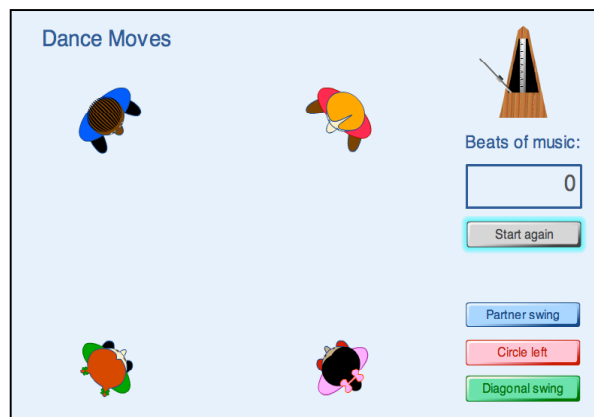
**Introduce the microworld to the class**

**5 minutes**

**Spirolaterals**



**Dance Moves**



Each lesson needs very little in the way of introduction. Give each pupil a copy of the appropriate handout and explain the purpose of the lesson:

*The aim of today's lesson is for you to explore a simple computer program.  
Your task is to try to answer these questions:*

- *What does the program do?*
- *What interesting problems does it suggest to you?*
- *Can you solve any of these problems?*
- *Can you make any conjectures and prove them?*

Explain how pupils are expected to work:

*I want you to work in pairs to see how the software works.*

*Try to record precisely what happens when you enter different numbers/ press different buttons.*

*As you do this, begin to think of some possible problems to investigate.  
You might, for example make up problems that start with the words:*

**Spirolaterals**

*"How can we make the computer draw ....?"*

*"What will happen if we .....?"*

**Dance Moves**

*"How many different ways...?"*

*"How can I make the dancers move so that...?"*

*"How many beats would....?"*

*"Can I get the dancers to do .... in .... beats?"*

*I'll ask you to share your ideas for problems with the rest of the class in five minutes!*

Issue pupils with clipboards, 1 cm squared paper, pencils, and rulers. Some pupils have also found it helpful to use coloured counters when working on Dance Moves.

**Pupils explore the microworld and generate problems****10 minutes**

Allow pupils 5 minutes to explore what happens as they type numbers into the software. Go round, encouraging pupils to describe what is happening precisely.

**Spirolaterals**

*So tell me how the computer knows what to draw when you enter those numbers.*

**Dance Moves**

*Tell me exactly how the dancers will move if you press diagonal swing?*

*Can you show me with counters?*

Encourage pupils to write down some ideas for problems to explore.

**Pupils describe what is happening and suggest problems to explore****15 minutes****Describing what is happening:****Spirolaterals**

*If I enter these three numbers (1, 2, 3) what will it draw when I press go?*

*What is the computer doing with these three numbers?*

**Dance Moves**

*If I press "Circle right" what do the dancers do?*

*What angle do they turn through?*

*Can you give us a demonstration?*

Encourage pupils to describe what is happening as clearly and fully as they can. For example, they might say:

**Spirolaterals**

*You start by facing to the right.*

*You move 1 unit forward then turn left 90°*

*You move 2 units forward then turn left 90°*

*You move 3 units forward then turn left 90°.*

*You move 1 unit forward then turn left 90°.*

*... and so on repeating 1,2,3 until you get back to the start.*

Repeat this process with other numbers until you think pupils know how the program draws the shapes.

### **Dance Moves**

The dancers each move forward and join hands to form a 'square'.  
They then rotate through  $270^\circ$  anticlockwise.  
They then step backwards.

Pupils may like to come out to the front and give a demonstration! This isn't as easy as it sounds for some as the orientation of the moves can confuse them.

### **Suggesting problems to explore**

Now brainstorm possible problems to explore and list some on the board or flipchart. Pupils might suggest ideas such as the following:

#### **Spirolaterals**

- *What will happen if we enter a single number and press "Go".  
2 numbers? 3 numbers? 4 numbers?  
Can we predict the types of shapes we will get?*
- *What happens when we change the order of the numbers?  
So how is (1,2,3) different from (1,3,2)?*
- *Do the shapes always go back to the start?  
When do they? When don't they?  
How can we predict this from the numbers?*
- *When do the shapes have rotational symmetry?  
Can we predict this from the numbers?*
- *When do the shapes have line symmetry?  
Can we predict this from the numbers?*
- *What happens if we enter the same number more than once?  
What happens with 3 numbers, like (1,1,2); (3,2,3)?  
What about 4 numbers, like (1,3,4,4)...?*

#### **Dance Moves**

- *How can we get the dancers to do a dance and then back to their original positions?  
What different ways are there of doing this?  
How many different numbers of beats are needed?*
- *What other different positions can the dancers end up in?  
How can we represent these positions?  
How can the dancers reach these different positions?*
- *Are there impossible configurations?  
How can we be sure that they are impossible?*
- *Can we find a dance that lasts, say, 64 beats so that the dancers end up where they started?*

Ask pupils to choose a particular problem to work on. Encourage them to be systematic as they try to answer their problem.

Discuss how they should record their work.

*When you think you have some hypotheses or conjectures, I want you to be able to show me the evidence for this. So as you work, try to keep careful notes of what you try. You may like to copy some of the diagrams by taking screen shots and pasting them into a word processor.*

### Pupils work on the problems

20 minutes

As pupils work on the problem, prompt them to think strategically and analytically:

*Can you state your problem to me clearly?  
What examples have you tried so far?  
What are you keeping fixed? What are you changing?  
Can you do this in a systematic way?*

*What have you found out so far?  
Can you see any patterns or relationships here?  
Can you explain **why** your idea seems to work?*

*How are you keeping a record of your work?  
Can you use a helpful notation?  
Why do you need to do this?  
Can you use the computer to do this?*

### Pupils report back and share findings

10 minutes

When most pupils have made significant progress with their problem, invite a few pairs to come to the front and communicate their ideas to the rest of the class. They can show some patterns on the software itself, projected for others to see. It does not matter if some have not yet reached any conclusions. They can still share their approaches and ideas.

*Let's stop and share some of the different approaches we have used and consider what you have discovered.*

*Tell us about:*

- *the problem you are solving;*
- *how you have organised your work;*
- *any conclusions you have reached so far.*
- *any explanations you have for your answers?*

As pupils share their ideas, ask others to contribute suggestions, further examples or counterexamples and ideas of what to do next.

For homework, pupils could be asked to write an account of their discoveries.



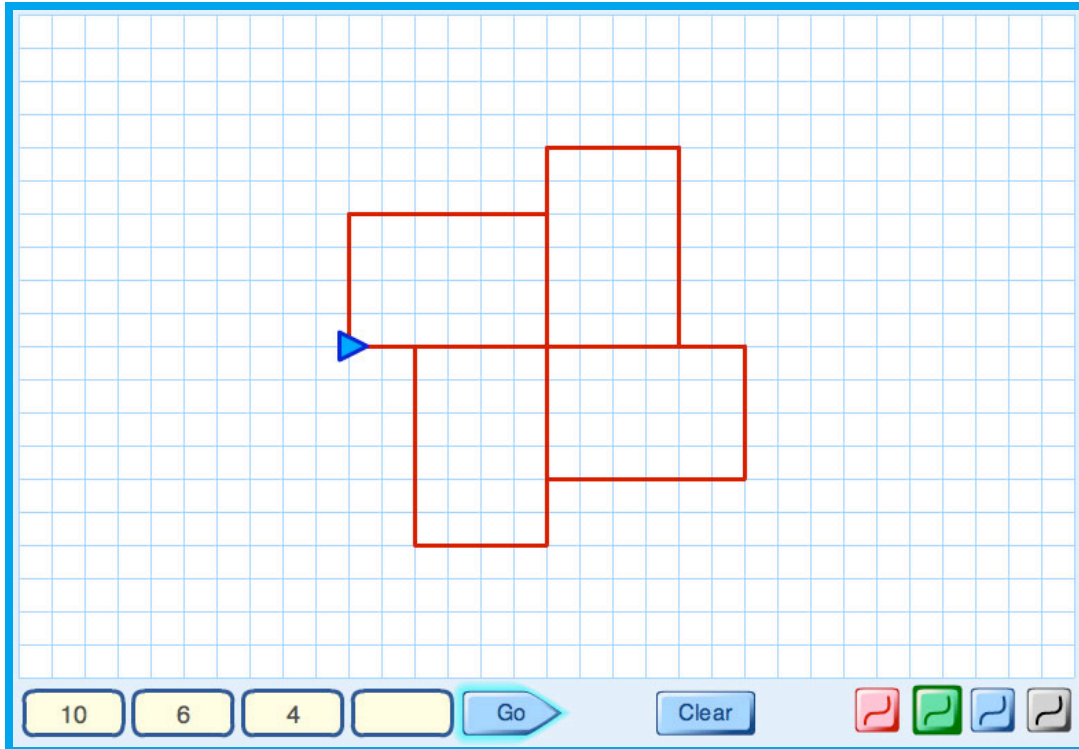






## 1 Two microworlds to explore

## Spirolaterals



Type some numbers into the *Spirolaterals* machine.  
Press "Go" and watch what happens.

- How do the numbers control what is drawn on the screen?
- List some questions to explore.  
These might start like this:  
"How can we make the computer draw ....?"  
"What will happen if we ....."

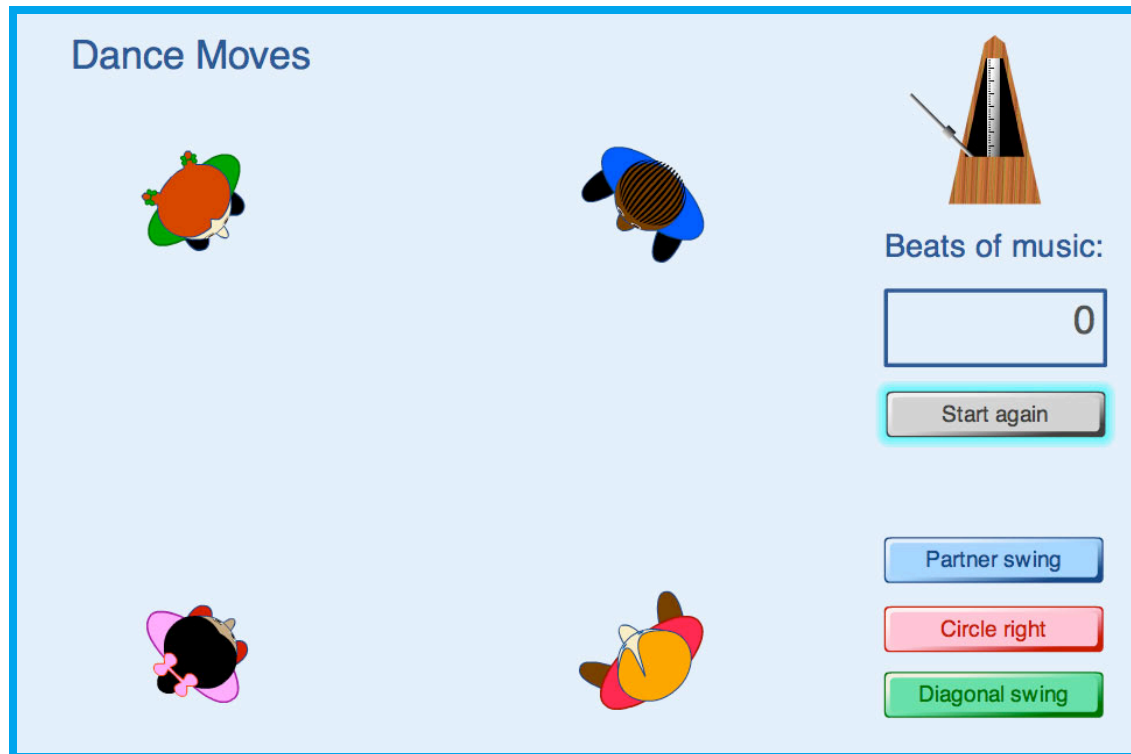
Try and answer your own questions!

- Make up some conjectures. These might start like this:  
" When you use three numbers then ..."  
" When you repeat a number then ..."  
" If you change the order of the numbers then ..."

Can you *prove* your conjectures?

## 1 Two microworlds to explore (continued)

## Dance Moves



Try clicking the different buttons on the software:

- Describe, as precisely as you can, what each button does.
- Write down some problems that occur to you.  
These might start like this:  
"How many different ways...?"  
"How can I make the dancers move so that...?"  
"How many beats would....?"  
"Can I get the dancers to do .... in .... beats?"

Now try to solve your own problem.

- Make up some conjectures.  
These might start like this:  
" You can make the dancers reach every position in ..."  
" It is impossible to ..."

Can you *prove* your conjectures?



### 3 A checklist for using ICT

*Please don't be offended if the following advice seems obvious and patronising - we all suffer from occasional 'common sense failure' when confronted with ICT!*

#### Choose appropriate ICT resources

- How will ICT contribute to the learning?
- Will the ICT be used as a thinking tool, a microworld or as a didactic tool?
- Who will be choosing whether or not to use the ICT?
- Which ICT resources will be available: an interactive whiteboard, data projector, graphical calculator projected on to a whiteboard, one standalone computer, a suite of computers, or a class set of graphical calculators?
- What software can I choose from?

Moving the whole class to an ICT lab is disruptive – and inevitably moves the focus of a lesson away from the maths and towards the hardware. Ideally, pupils should come to see ICT as part of their everyday mathematical toolkit. This is not easy, given limited resources.

- If you *do* have computers, graphic calculators or other ICT resources available in the mathematics classroom, try to ensure that they are available to any pupil (who can make a reasonable case for their use) when working on unstructured problems. Don't just wheel them out for lessons which “need” them – this applies to other, non-ICT resources as well.
- If pupils have home PCs but are prevented from using them for maths because of the cost of suitable software (such as spreadsheets or interactive geometry), remember that there are often good, free alternatives available.

#### Prepare the room

- How will the ICT be used? Who will operate it?
- How will you arrange the room so that pupils have the space to discuss, and record their work, and so that you can monitor what they are doing?

Some school computer suites leave little desk space for writing – yet many ICT-based investigations are almost impossible without paper for notes and rough work. When recording, especially when using mathematical symbols and diagrams, word processors are no substitute for clipboards and paper.

#### Test the ICT resources

- Have you installed and tested the software on *pupils'* computers – not just your home machine or one reserved for staff use?
- If your school has a network, have you made sure that your software works when you are logged in as a *pupil*?
- What replacement activity will you use if there is, for example, a power cut?

Check the system requirements for your software, and whether anything needs to be installed before use. Some software requires “installation” to ensure that files are copied into specific places – you might need IT support to achieve this on a school network or computers with strict security settings.

Many educational “microworlds” and “applets” – can be run online, or just by opening a file, but they may also require specific “plug-ins” or “players” such as Flash Player, Adobe Reader, Java or QuickTime. These are usually free downloads – but will need installing. Even if you already have them, there may be a newer version available – worth checking if you are having problems.

Some “interactive” web sites – or other software that runs in a web browser can be blocked by security settings on your computer. Usually you will be offered an “allow blocked content” button – or similar.

### Plan the types of output required

- How will pupils save their work?
- How will pupils share their work with yourself and others?  
Can they project their work so that others may see and discuss it?  
Will they be able to use presentation software?

Both you and the pupils will want some record of their work. In some cases, this may just be written notes, in others a completed document in a word processor or spreadsheet. Some software might generate custom reports or printouts. Otherwise: Do pupils know how to capture “screen shots” and paste them into a word processor or presentation program to record their work? Saving files for later use can be a problem on school computers – find out from your IT support staff what the rules are at your school and make sure pupils follow any file naming conventions and save things in the correct place. If pupils are printing out to a shared printer, remind them ad nauseum to make sure that everything they print includes their name, and allow sufficient time for this.

### Plan the lesson itself=

- How will you group pupils?
- How and when will you work with the whole class?  
How and when will you work with small groups/ individuals?
- How will you support/challenge pupils who need extra help/find the work too easy?
- How will you discourage blind trial-and-error approaches?
- How will you enable pupils to present and share their work?

Time in the ICT lab will always be limited – try and make the most of it.

- Avoid taking time on “housekeeping” (collecting/returning homework, announcements) at the start or end of the lesson – can it wait?
- Try and ensure that the *previous* lesson is “wrapped up” on time so it doesn’t spill over into the ICT time.

As soon as pupils get into the ICT room their attention will be drawn to the computers, so it is not a good environment for conventional teaching. Some Bowland case studies have one or more conventional lessons as preparation for an ICT session – allow *plenty* of time for this the first time you try the case study.

For further advice on planning to use ICT, we suggest reading the DCSF publication: *Integrating ICT into mathematics in Key Stage 3: 2003*. This is available from:

[http://www.standards.dfes.gov.uk/secondary/keystage3/all/respub/ma\\_integrate\\_ict](http://www.standards.dfes.gov.uk/secondary/keystage3/all/respub/ma_integrate_ict)



## 4 Making and selling a magazine

Some teenagers want to raise money by making and selling a new magazine.

They conduct a survey to find out how the selling price might affect the number of people that will buy it.

They ask the following question to 100 people:

**"How much would you be prepared to pay for this magazine?"**



This is what they find:

Selling price (£)	0	0.50	1.00	1.50	2.00	2.50
Number of people who would buy it at this price.	100	82	60	42	18	0

Each magazine costs 10 pence to make.

What should the selling price be in order to make the most money?

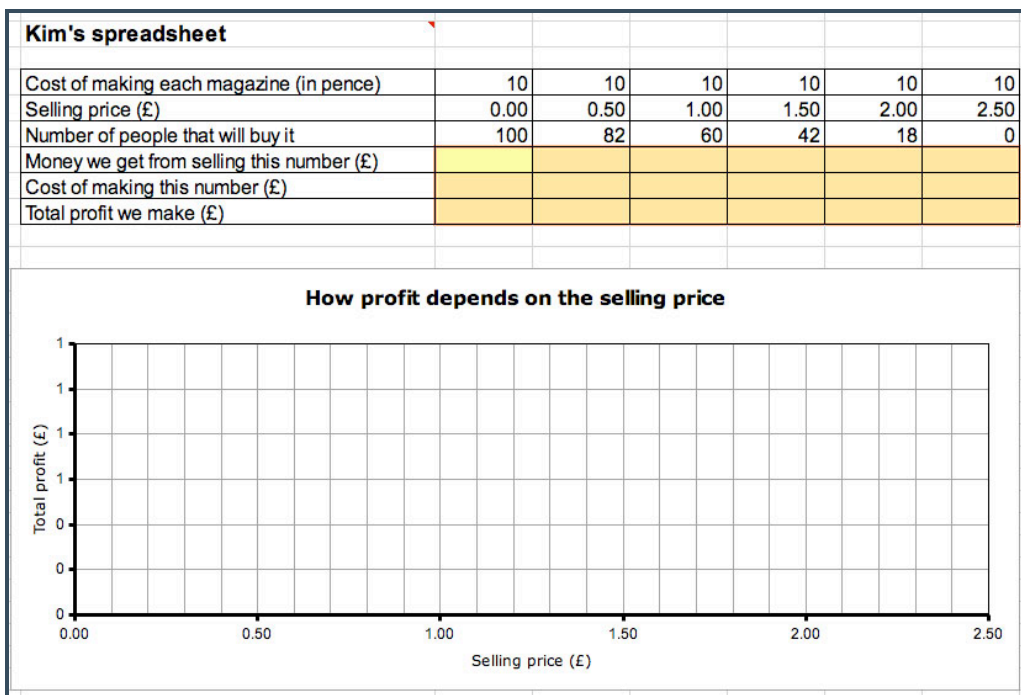
Suppose production costs increase... how will this affect your answer?

### Using a spreadsheet

Kim has started to solve this problem with a spreadsheet.

See if you can work out what she has done and use the spreadsheet to solve the problem.

(Start by changing Kim's name to your own!)



## 5 Spreadsheet Hints

### 1. Using a formula

For example: Area = Length x Width

	A	B	C	D
1				
2	Length	6	10	16
3	Width	4	8	9
4	Area	=B2*B3		

This is cell **B2**

This is cell **B3**

Multiply the number in **B2** by the number in **B3**

Press Enter

	A	B	C	D
1				
2	Length	6	10	16
3	Width	4	8	9
4	Area	20		

Double-click to see the formula again

\* means **multiply**  
/ means **divide**

### 2. Copy a formula across rows...

	A	B	C	D
1				
2	Length	6	10	16
3	Width	4	8	9
4	Area	20		

Drag...

	A	B	C	D
1				
2	Length	6	10	16
3	Width	4	8	9
4	Area	20	80	144

See how the formulae change:

=C2\*C3

=D2\*D3

This is the quickest way!  
You can also use Cut & Paste or the "fill" command...

### 3. Advanced: naming cells

You might not have done this in ICT...

Price of carpet = Price per square m x Area

	A	B	C	D
1	Price per square m (£)	10		
2	Length	6	10	16
3	Width	4	8	9
4	Area	20	80	144
5	Price of carpet (£)	200	0	0

You type  
= B4 \* B1  
and drag it across...

What?!!

=C4 \* C1  
should be:  
=C4 \* B1

**Beware!** Sometimes you want some bits of the formula to stay the same when you copy it!  
  
If you want lots of formulae to refer to the same cell you can give the cell its own name:

2. Change this from **B1** To **price**

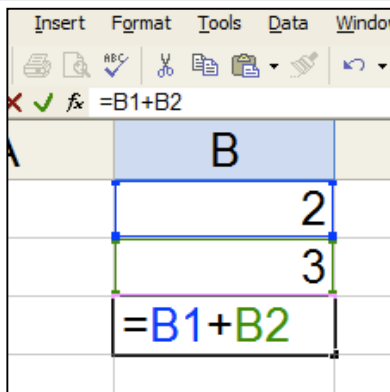
	A	B	C	D
1	Price per square m (£)	10		
2	Length	6	10	16
3	Width	4	8	9
4	Area	20	80	144
5	Price of carpet (£)	200	800	1440

1. Select the "Price" cell

3. Change to = B4 \* price and drag along...

Now if you change the number in B1 everything will change to match...

### 4: Teachers: using spreadsheets on a whiteboard



Choose "View -> Zoom" and experiment so that the spreadsheet is big enough to read easily.

You can also try "View -> Full Screen" to make the most of the screen.

**Ignore the formula bar if it is too small:** double-click on a cell to enter/edit a formula.

(Tools -> Options -> Edit -> Edit directly in cell if it doesn't work)















## 8 Suggested further reading

*A 200 year old problem that can lead pupils to high levels of pupil thinking and activity.*

Leadbetter M (2007) 'More than ladders' *Mathematics Teaching*, 207, pp 9-11

<http://www.atm.org.uk/mt/archive/mt204files/ATM-MT204-09-11-mo.pdf>

*An investigation of what teachers see as the issues on teaching mathematics with ICT.*

Hennessy, S., Ruthven, K., & Brindley, S. (2005). Teacher perspectives on integrating ICT into subject teaching: Commitment, constraints, caution and change. *Journal of Curriculum Studies* 37 (2) 155-192.

Available online at <http://dx.doi.org/10.1080/0022027032000276961>

*What disaffected pupils can learn with ICT.*

Papert, S and Harel I. 'Situating Constructionism' in Harel, I. and Papert, S. (1991) *Constructionism*, Norwood, NJ: Ablex Publishing Corporation

Available online at <http://www.papert.org/articles/SituatingConstructionism.html>

*Using microworlds in learning mathematics.*

Healy' L. (2008) 'Charting the microworld territory: the placing of theoretical signposts Symposium on the Occasion of the 100th Anniversary of ICMI

Available online at <http://www.unige.ch/math/EnsMath/Rome2008/WG4/Papers/HEALY.pdf>

*Coming right up to date with ICT.*

Oldknow A (2008) 'It's 2008 - So what you got to offer, then? - Using ICT to put learners in touch with mathematics'.

Available online at <http://www.mei.org.uk/files/pdf/BectaArticleAOV2.pdf>

*An overview of 'best practice' with ICT.*

Oldknow, A (Ed) (2005) *ICT and Mathematics: A Guide to teaching and learning Mathematics 11-16 using ICT*, The Mathematical Association, Leicester